

$$\hat{\beta}_*: \cdot: H_1(\hat{D}_n) \longrightarrow H_1(\hat{D}_n)$$